

# ROCK SLOPE STABILITY ANALYSIS USING BLOCK THEORY AND PROBABILISTIC APPROACH: AN APPLICATION AT NATIONAL ROAD NO.6, VIETNAM

QuocPhi Nguyen<sup>1</sup> and TruongThanh Phi<sup>2</sup>

<sup>1</sup>Department of Environmental Sciences, Hanoi University of Mining and Geology, Vietnam  
Email: nguyenquocphi@humg.edu.vn

<sup>2</sup>Institute of Marine Geology and Geophysics, Vietnam Academic of Science and Technology, Hanoi, Vietnam  
Email: thanhgislab@gmail.com

## ABSTRACT

*Rock slopes are the major concern of stability of roads constructed in mountainous terrain and spatial visual of rock blocks in real shape and size in space was normally not available from conventional slope stability analysis. The stability of rock slopes is mainly dependent on geological characteristics such as discontinuities inside the rock mass. In this study, the stability of rock slope is conducted in two steps: First, the unstable blocks are identified using Block theory and then, the potential failure of rock blocks are analysed using probability approach. The application of Block Theory helps finding potential failure rock blocks (key blocks) and then, their location, volume and geometry (shape, size) can also be identified on slope surface. However, there are many factors can affect the stability of rock slopes, such as the orientation and dip angle of slope face and fractures, the cohesion and friction angle of discontinuities, the strength properties, etc, and the uncertainty of input parameters were considered as data ranges for probability analysis. The final factor of safety (FS) of each failure mode is calculated using limit equilibrium method. In order to verify the developed procedure, the field survey data of rock slopes along national road No.6 were analysed. Results could significantly help the field engineer to decide which portion of slope need immediate supports. The integrated approach provides a mathematical formulation that enables the engineer to relate the rock structures to the excavation surface and analyse the stability of rock slope under probability functions of collected data.*

## 1. INTRODUCTION

The analysis of rock slope stability mainly depends on the correct interpretation of the structural geology, the identification of potential unstable blocks which can be released by the creation of the excavation and the analysis of the reinforcing forces required to stabilize these blocks.

The traditional stereographic method has the advantages at the preliminary stage of an excavation. In this stage, detail measured data are not available; therefore the approach can provide a solution for the designing of the excavation. The approach makes assumption on fracture data representation as fracture sets. Therefore, if the joint set orientation or spacing is dispersed with some central tendency, the analysed results vary a lot from reality. Therefore, stereographic method can only gives the overall idea of how the unstable blocks will look like on the excavation faces.

As the excavation approach into next stage, the analytic information from stereographic method basically does not answer the critical question of support design where the possible key blocks locate. The size and property of the specific block generated in integrated

procedure of Block theory and probabilistic approaches can overcome this situation and are more suitable for support design. Field data from Dong Bang area, Mai Chau, Hoa Binh along national road No.6, from Km134+240 to Km139+600 are used to illustrate the proposed procedure.

## 2. METHODOLOGY

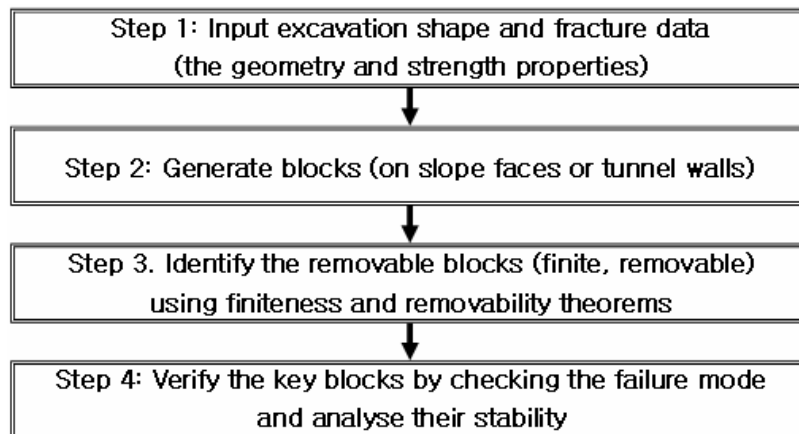
### 2.1 Block Theory

The main aim of the Block Theory (Goodman and Shi, 1985; Goodman, 1995) is to geometrically analyze the distribution of different types of blocks in relation to discontinuity surfaces and to determine under which conditions these blocks may lose stability (from a kinematic point of view).

Since rockmass composes of fractures and intact rock pieces, the fractures in the rockmass intersect each other and divide rockmass into rock blocks of various shapes and sizes. When the rock-mass is excavated, the balance stability of the rockmass could be disturbed. Once the rock blocks start sliding, a chain reaction may occur after that and it may lead the whole slope to collapse.

The geometry of fractures and excavation shape are defined in real 3D space from detail survey. The exact position, size and shape of each fracture are known in advance, together with the parameters that define its strength, such as cohesion and friction angle. The positions of all surfaces are fixed that make only one specific arrangement of the block assembly and from this, the exact size of each block can be identified (Nguyen, 2009; Phi, 2012).

The following part will describe the principles of block theory in a brief manner of specific approach. The stability of resulting blocks will be evaluated at local conditions for discontinuous rock slopes. The steps of analyzing can be concretized as:



**Figure 1. Steps of key block analyses.**

The idea of Block theory is shown in details in Goodman and Shi (1985), however, two theorems of finiteness and removability are the major part of Block theory could be illustrate on stereonet.

### ***Theorem of finiteness - The determination of key block***

On stereonet, the space above excavation surface is called free space (denoted as space pyramid, SP) and the space below the excavation surface is called excavation space (denote as excavation pyramid, EP). The spherical regions formed by the intersection of the block faces are known as Joint Pyramids (JP) and the block pyramid (BP) is the intersection of the JP and the EP.

According to the ***finiteness theorem***, a block is finite if its block pyramid (BP) is empty ( $\phi$ ):

$$BP = JP \cap EP = \phi \quad (1)$$

Or in other words, Joint Pyramids (JP) is fully within Space Pyramid (SP):

$$JP \subset SP \quad (2)$$

The blocks do not satisfy criterion (2) fall into the category of infinite blocks. Therefore, by plotting block faces on a stereonet and using the criterion given in equation 2, the finite blocks can be separated from the infinite blocks.

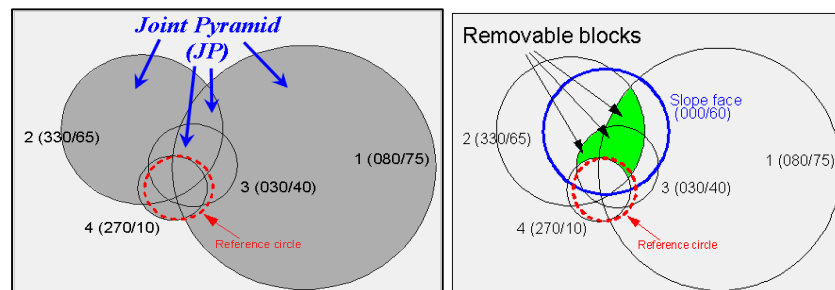
### ***Theorem of removability – Analyse the block movement***

Once the block is identified as finite, it could be either removable or non-removable (tapered) block. According to the ***removability theorem*** (Goodman and Shi, 1985), the removable block must not only satisfy the criterion given in equation (2) but also the criterion of equation 4:

$$JP \neq \phi \quad (3)$$

Since the *JP* is empty in tapered (non-removable) block, it does not show on a stereographic projection as the emptiness of intersections of fracture orientations can not be seen on stereonet. Therefore, the joint pyramids that satisfy the criterion given in equation (2) on a stereographic projection belong to removable block categories.

Figure 2 shows an example of four joint planes and a slope face plotted on stereonet. The slope face is presented in blue color and assumes that the Space Pyramid (*SP*) is the area inside its great circle. A series of joint blocks (*JPs*) are defined by the four joints but only the three blocks (colored green) are entirely included in slope face circle (*SP*). Therefore, these three blocks corresponds to removable blocks by finiteness and removability theorems. All the other regions (blocks) are at least partly outside the *SP*; hence, accordingly they are non-removable.



**Figure 2. Identification of removable blocks on stereonet.**

Therefore, the conditions of each block types can be summarized as follow:

**Table 1. Block type and corresponding conditions.**

| Block type      | Conditions                        |
|-----------------|-----------------------------------|
| Infinite block  | $JP \neq \phi$ and $BP \neq \phi$ |
| Tapered block   | $JP = \phi$ and $BP = \phi$       |
| Removable block | $JP \neq \phi$ and $BP = \phi$    |

***Kinematic analysis - Find out the failure modes***

Removable blocks require further analysis. There are three basic failure modes: Falling, sliding on a single plane, and sliding on the intersection of two planes. A free surface of excavation is needed for a block sliding, and the excavation surface can not be the sliding surface.

**2.2 Stability of rock block by probabilistic approach**

The probabilistic approach normally assumes that the geometrical and mechanical parameters of fractures following a statistical distribution model (Park and West, 2001). The failure probability is then evaluated by Monte Carlo simulation. The method solves mathematical problems through random sampling and repeated calculation through predefined input data ranges.

If the Block theory result indicates that the movement is possible, the stability of the block can be evaluated through the factor of safety using limit equilibrium analysis. The factor of safety in plane failure is evaluated simply by the following equation (Hoek and Bray, 1981):

$$FS = \frac{cA + W \cos \alpha \tan \phi}{W \sin \alpha} \quad (4)$$

Where:  $W$  is the block weight force acts vertically downward  
 $\alpha$  is the angle of sliding surface  
 $A$  is the base area of the block  
 $c$  and  $\phi$  are the cohesion and friction angle, respectively

For wedge block, the stability analysis can be performed using a procedure described by Hoek and Bray (1981):

$$FS = \frac{3}{\gamma H} (c_A X + c_B Y) + (A - \frac{\gamma_w}{2\gamma}) \tan \phi_A + (B - \frac{\gamma_w}{2\gamma} Y) \tan \phi_B \quad (5)$$

Where:  $c_A$  and  $c_B$  are the cohesive strengths of planes A and B  
 $\phi_A$  and  $\phi_B$  are the angles of friction on planes A and B  
 $\gamma$  and  $\gamma_w$  are the unit weights of rock and soil respectively  
 $H$  is the total height of the wedge

The dimensionless factors  $X$ ,  $Y$ ,  $A$  and  $B$  depend upon the geometry of the wedge:

$$X = \frac{\sin \theta_{24}}{\sin \theta_{45} \cos \theta_{2,na}} \quad (6)$$

$$Y = \frac{\sin \theta_{13}}{\sin \theta_{35} \sin \theta_{1,nb}} \quad (7)$$

$$A = \frac{\cos \psi_a - \cos \psi_b \cos \theta_{na,nb}}{\sin \psi_5 \sin^2 \theta_{na,nb}} \quad (8)$$

$$B = \frac{\cos \psi_b - \cos \psi_a \cos \theta_{na,nb}}{\sin \psi_5 \sin^2 \theta_{na,nb}} \quad (9)$$

Where:  $\psi_a$  and  $\psi_b$  are the dips of planes A and B respectively  
 $\psi_5$  is the dip of the line of intersection.

### 3. CASE STUDY: SLOPE STABILITY AT ROCK SLOPES ON NATIONAL ROAD NO.6, VIETNAM

#### 3.1 Geology and fracture survey

National road No.6 is the road connects Hanoi capital to the northwest provinces of Vietnam. Total length of the road is 504km starting from Hanoi to Dien Bien which has a series of mountain pass over 10% slope and is known as the route of many potential slope failures. The studied area is located in Dong Bang, Mai Chau district of Hoa Binh province, from Km134+240 to Km139+600, between 20°41'58" and 20°43'35" in latitude and 105°00'41" and 105°03'34" in longitude. Topographic map and geological map as well as the layout of road section are provided in figures 3 and 4.

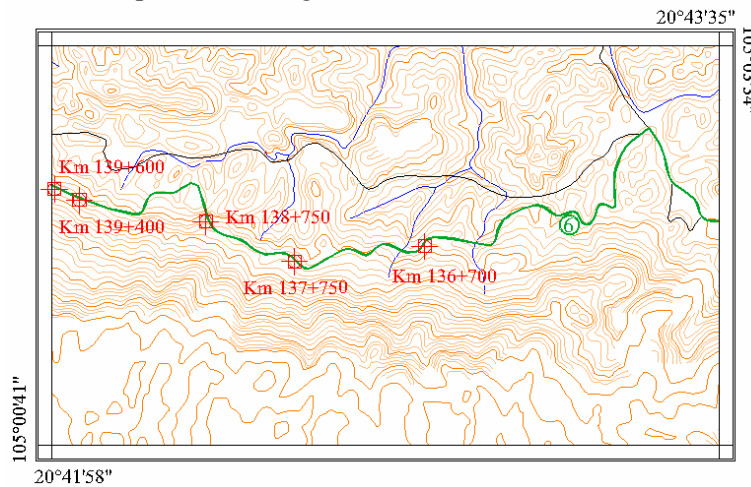
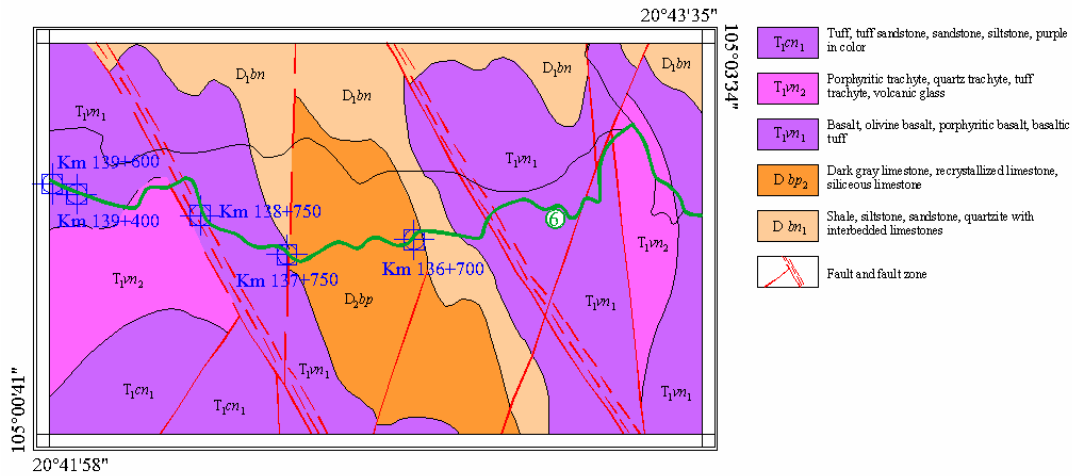


Figure 3. Topographic map of study area and slope locations



**Figure 4. Geological map of study area**

The study area is composed chiefly of highly foliated limestone and marl of Ban Pap formation (Devonian); shale, siltstone and sandstones of Ban Nguon formation (Devonian); basaltic rocks and tuff, tuff sandstones of Vien Nam formation (Triassic). All studied slopes located on the left side of the road and the rock are slightly to highly weathered at exposed surface (table 2).

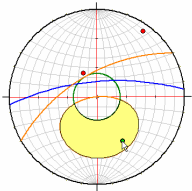
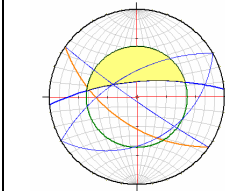
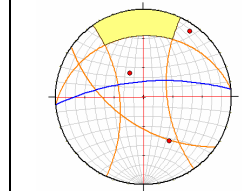
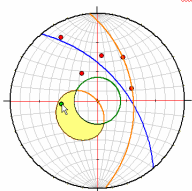
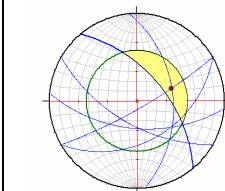
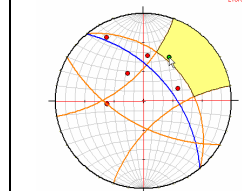
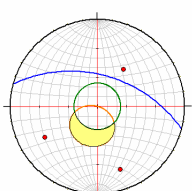
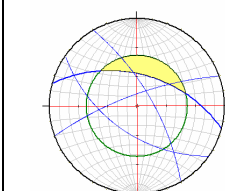
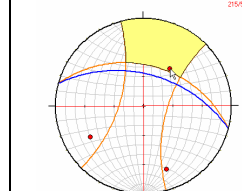
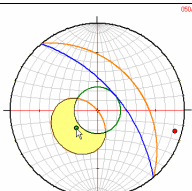
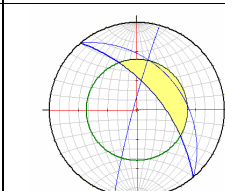
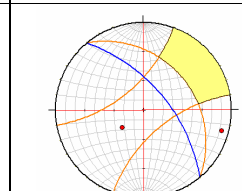
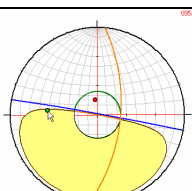
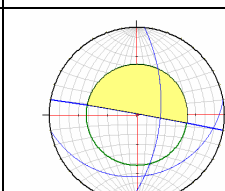
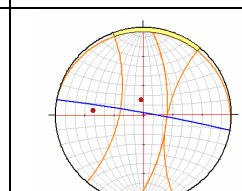
**Table 2. Geological and fracture survey results**

| Station   | Slope face | Rock type | Fracture properties |             |              |            |                  |
|-----------|------------|-----------|---------------------|-------------|--------------|------------|------------------|
|           |            |           | Type                | Orientation | Roughness    | Weathering | Filling material |
| Km136+700 | 355/70     | Limestone | Bedding             | 150/35      | Smooth       | Highly     | Cohesive soil    |
|           |            |           | Joint               | 330/60      | Rough        | Moderately | -                |
|           |            |           | Joint               | 215/85      | Rough        | Slightly   | -                |
| Km137+750 | 050/55     | Limestone | Fault               | 085/45      | Rough        | Moderately | Rock fragments   |
|           |            |           | Bedding             | 150/40      | Slightly     | Slightly   | Cohesive soil    |
|           |            |           | Joint               | 185/55      | Smooth       | Slightly   | -                |
|           |            |           | Joint               | 210/60      | Rough        | Slightly   | -                |
|           |            |           | Joint               | 150/80      | Very rough   | Highly     | Clay             |
| Km138+750 | 015/50     | Basalt    | Joint               | 215/55      | Slickensided | Slightly   | -                |
|           |            |           | Joint               | 060/70      | Rough        | Highly     | Cohesive soil    |
|           |            |           | Joint               | 340/75      | Very rough   | Highly     | Rock fragments   |
| Km139+400 | 050/60     | Basalt    | Joint               | 050/35      | Smooth       | Moderately | Cohesive soil    |
|           |            |           | Joint               | 285/85      | Slightly     | Highly     | Rock fragments   |
| Km139+600 | 010/88     | Basalt    | Joint               | 170/20      | Slightly     | Moderately | Cohesive soil    |
|           |            |           | Joint               | 095/60      | Slightly     | Highly     | Rock fragments   |

### 3.2 Block investigation and stability analysis

Potential key blocks are identified using Block theory and then, for every potential block, kinematic analyses were conducted using the collected major discontinuities of slopes to estimate possible failure type of plane, wedge and toppling failures. A computer code, PCUNet, was developed for stereonet plots and calculates the potential failure modes (table 3).

**Table 3. Kinematic analysis of rock blocks**

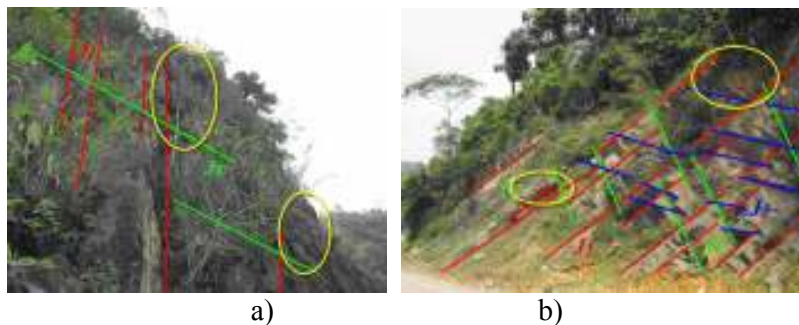
| Station                  |                   | Failure modes   |  |   |
|--------------------------|-------------------|---|--|---|
|                          |                   | <i>Planar</i>   | <i>Wedge</i>   | <i>Toppling</i>   |
| <b><i>Km 136+700</i></b> | Stereonet plot    |    |    |    |
|                          | Failure potential | Unstable (330/60)   | Stable   | Stable  |
| <b><i>Km 137+750</i></b> | Stereonet plot    |    |    |    |
|                          | Failure potential | Unstable (085/45)   | Unstable (150/80 & 085/45)   | Unstable (210/60)   |
| <b><i>Km 138+750</i></b> | Stereonet plot    |  |  |  |
|                          | Failure potential | Stable  | Stable   | Unstable (215/55)   |
| <b><i>Km 139+400</i></b> | Stereonet plot    |  |  |  |
|                          | Failure potential | Unstable (050/35)   | Stable   | Stable  |
| <b><i>Km 139+600</i></b> | Stereonet plot    |  |  |  |
|                          | Failure potential | Unstable (095/60)   | Stable   | Stable  |

Random properties of fault, joint and bedding planes are assigned based on statistical analyses (mean and standard deviation) for slope and discontinuity orientations, cohesions and friction angles, roughness and shear strengths. All possible discontinuity combinations were generated and analyses for stability. The results of stability analyses are listed in table 4, indicating the factor of safety (FS) and sliding probabilities different modes of failure.

**Table 4. Stability analysis and probability of failure**

| Station   | Sliding blocks      | Sliding type | Weight of rock blocks (ton) | Failure plane(s) | Factor of safety (Mean $\pm$ Std.) | Sliding probability |
|-----------|---------------------|--------------|-----------------------------|------------------|------------------------------------|---------------------|
| Km136+700 | Key block           | Planar       | 1,252.8                     | 330/60           | 0.227 ( $\pm$ 0.35)                | 0.96                |
| Km137+750 | Potential key block | Planar       | 2,985.4                     | 085/45           | 0.430 ( $\pm$ 0.12)                | 0.62                |
|           | Key block           | Wedge        | 3,497.1                     | 150/80 & 085/45  | 0.513 ( $\pm$ 0.45)                | 0.77                |
|           | Key block           | Toppling     | 1,865.4                     | 210/60           | -                                  | 0.70                |
| Km138+750 | Key block           | Toppling     | 982.6                       | 215/55           | -                                  | 0.67                |
| Km139+400 | Key block           | Planar       | 11,895.3                    | 050/35           | 0.609 ( $\pm$ 0.27)                | 0.95                |
| Km139+600 | Potential key block | Planar       | 9,574.8                     | 095/60           | 0.247 ( $\pm$ 0.37)                | 0.72                |

Since the rock slope stability is mainly dependent upon the characteristics of discontinuities, the random properties of discontinuities play a critical role in probabilistic analysis of rock slopes. As the results shown, Block theory was successful in identifying potential failure blocks based on the geometry discontinuities and slope faces. Next, probabilistic analysis has been used to quantify and model the variability and uncertainty of input data, which are very commonly in geological data. In probabilistic analysis, input parameters are considered as random variables and their random properties are characterized using statistical parameters.



**Figure 5. Identified key blocks at a) Km137+750 and b) Km139+400**

The consequences of key block analysis and succeeding stability analysis for the determined key blocks allow economical design of rock reinforcement at next step. Factor of safety and sliding probability can provide the idea of which rock block need additional treatment first. For a particular excavation, these key blocks form specific sets of “danger zones” within the excavated rock mass.



#### 4. CONCLUSIONS

The experiment at slopes along national road No.6 shows the probability of failure is a conditional probability based on the precise location of unstable block. The integrated approaches of Block theory and probabilistic analyses are simple and easy to apply in the field by making simple codes with user-friendly interface. The Block theory has the advantage of locating the unstable key blocks on excavation faces (Um and Kulatilake, 2001). This helps the calculating of static rock block and supporting the design of rock structures in blocky rocks. It assists the determination of the location, dimensions and stability of individual blocks when the geometry of an excavation and the fractures is well characterized. The combination of Block theory and probability analyses could give more significant results for designing.

Another reason for Block theory will be attractive to rock engineering application are: (1) The analysis of Block theory is in three dimensions; (2) Out of many rock blocks created by the intersection of fractures, there are only few unstable blocks and the Block theory finds these “key blocks” in efficient way; (3) The Block theory only considers the orientation and the resisting strength of fracture planes but not the deformation of the blocks, therefore, it simplifies the stability analyses and could be effective when fast decisions for stability judgment is necessary.

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